



Mathematical Institute

Quantum field theory meets topology

Claudia Scheimbauer

Mathematical Institute
University of Oxford

Swiss National Science Foundation



Oxford Mathematics

About me

UNIVERSITY OF OXFORD
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Academic past

undergraduate → PhD → postdoc

TU WIEN → ETH zürich → Max Planck Institute for Mathematics

Non-academic

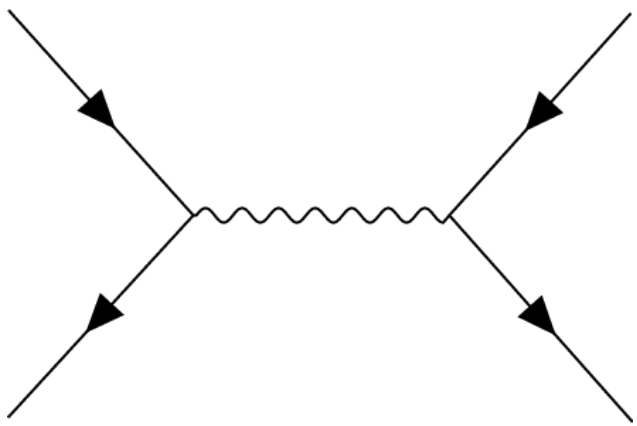




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Quantum field theory



Topology

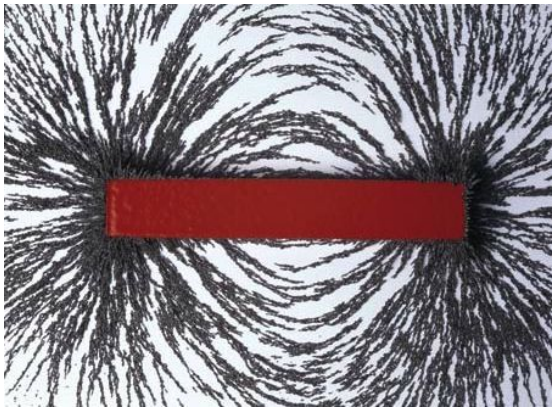


image: Sebastian Huber, ETH Zurich

Field theory

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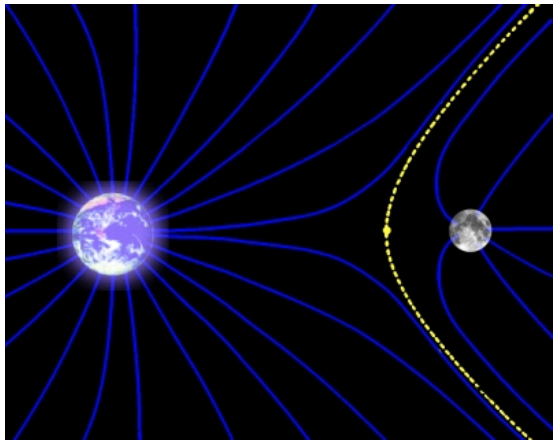
...classically, study *fields*, such as electro-magnetic, gravitational fields



images:

kullabs.com,

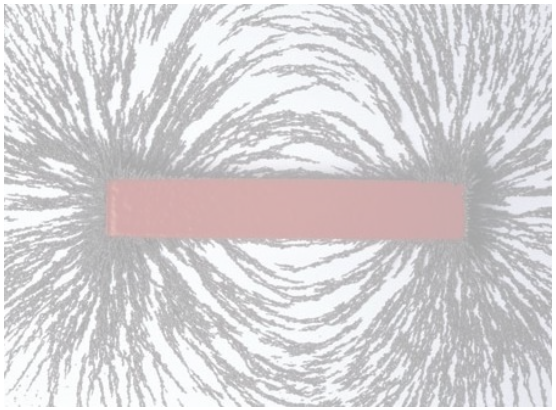
xkcd.com

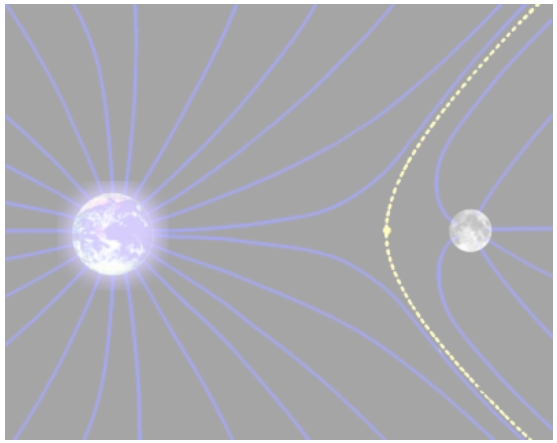


Field theory

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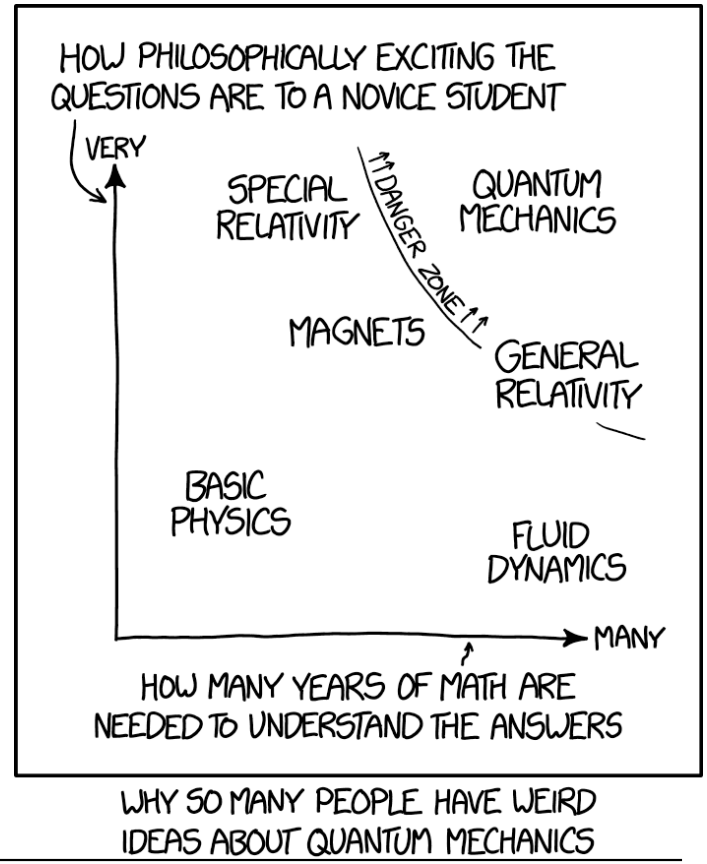


images:

kullabs.com,

xkcd.com

...*quantum* fields?



Topology

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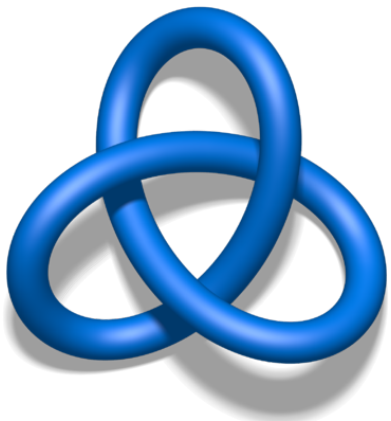
...study *spaces*, up to deformation ("homeomorphism")



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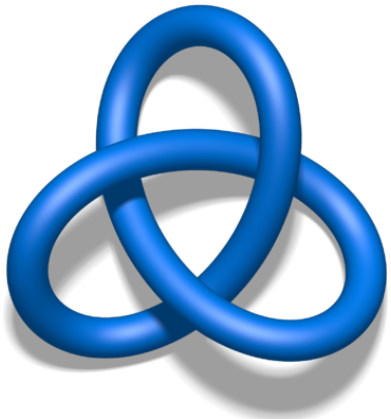




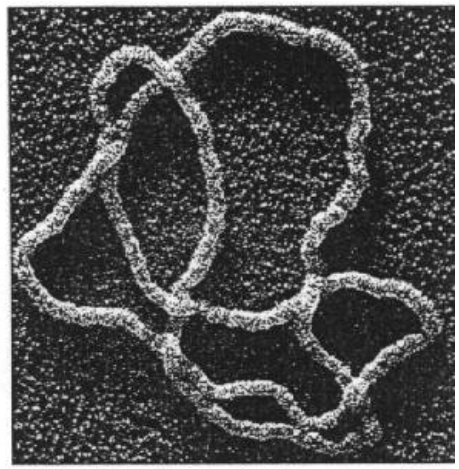
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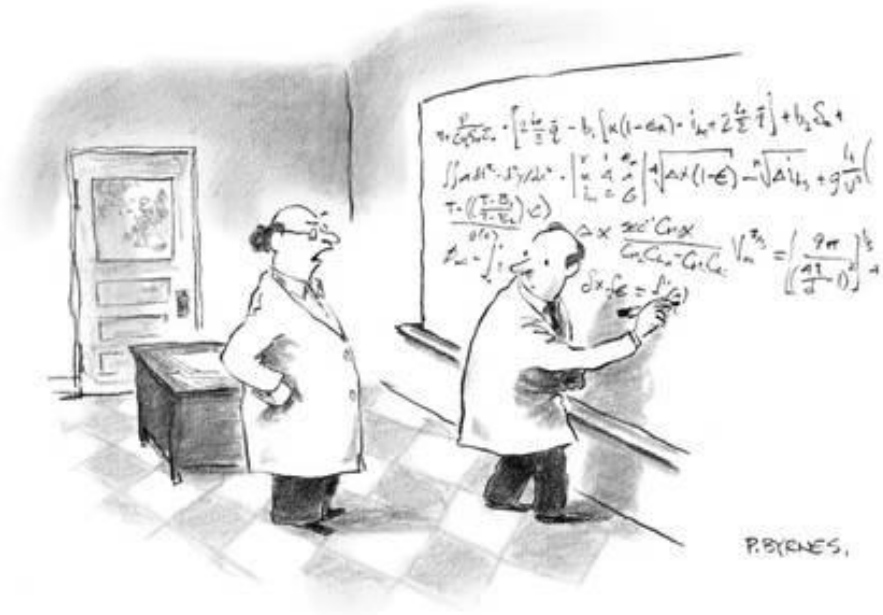




Physicists versus Mathematicians

Mathematical physics versus physical mathematics

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"Duh!"

One of the guiding principles is the goal of understanding the ultimate foundations of physics. [...] But, [...] it is also true that getting there is more than half the fun: If a physical insight leads to a significant new result in mathematics, that is considered a success. – G. Moore

The first meeting: Topological Field Theories

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Physics is very interesting: There are many, many interesting theorems. Unfortunately, there are no definitions. – *D. Kazhdan*

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Definition (1980's, Atiyah, Segal; after ideas by Witten)

A *2-dimensional topological field theory* is an assignment Z as follows:

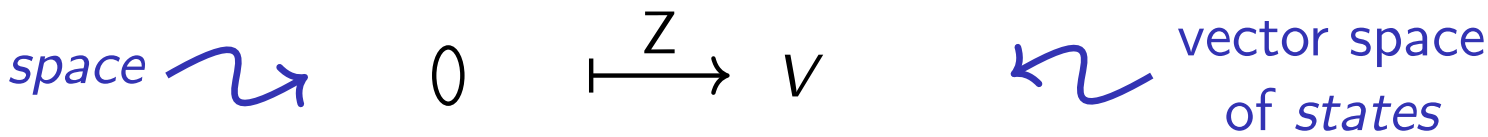
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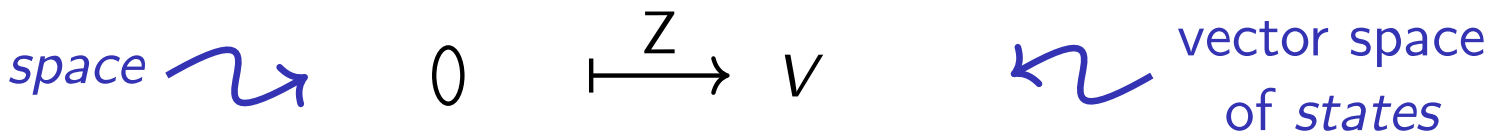


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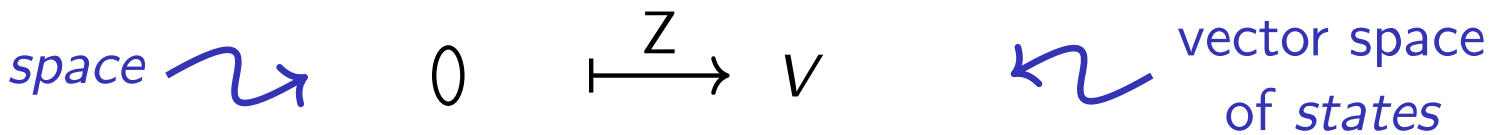


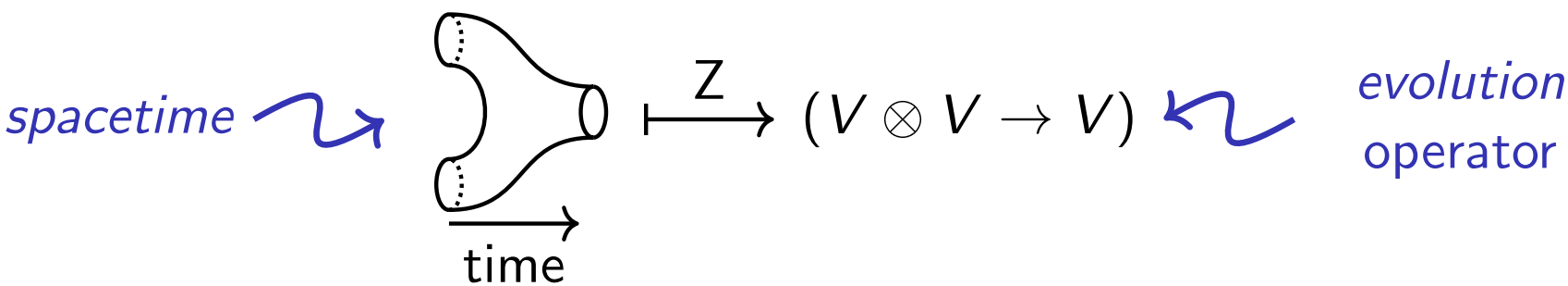
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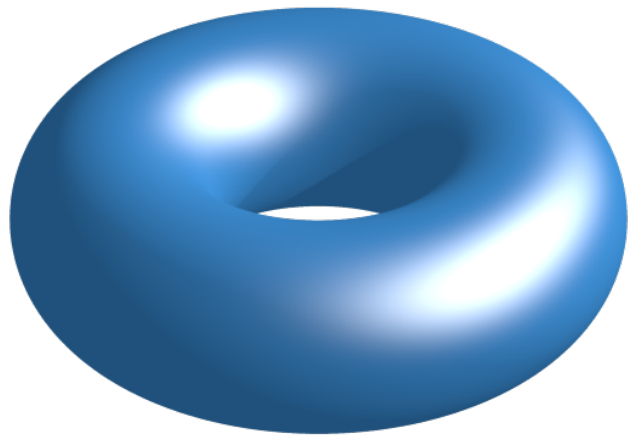
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extending TFTs (...using "higher categories")



An extended topological field theory is...

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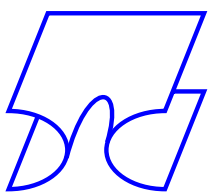
Spacetimes $\longrightarrow$ Vect

 $\longmapsto$ linear map

• • $\longmapsto$ vector space

An extended topological field theory is...

Spacetimes $\longrightarrow$ Vect ?


$\longmapsto$

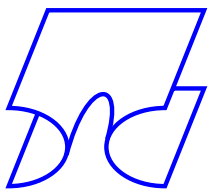

$\longmapsto$ linear map ?

• • $\longmapsto$ vector space

An extended topological field theory is...

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Spacetimes $\longrightarrow$ Vect $\quad 2\text{Vect} \overset{?}{=} \text{Alg}_1$


$\longmapsto$ homomorphism

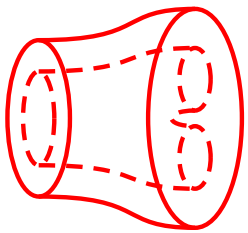

$\longmapsto$ linear map $\quad$ bimodule ?


$\longmapsto$ vector space $\quad$ algebra

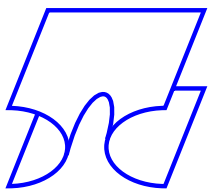
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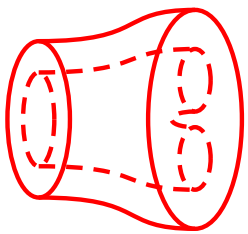
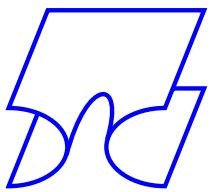
$\longmapsto$ linear map bimodule ?

?

• • $\longmapsto$ vector space algebra

An extended topological field theory is...

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Spacetimes $\longrightarrow$	Vect	2Vect $\stackrel{?}{=}$ Alg_1	3Vect $\stackrel{?}{=}$ Alg_2
 $\longmapsto$			homomorphism
 $\longmapsto$		homomorphism	bimodule of bimodules
 $\longmapsto$	linear map	bimodule ?	bimodule ?
 $\longmapsto$	vector space	algebra	E_2-algebra